MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY

GEOPHYSICAL SUPPLEMENT

Vol. 6 No. 7 1953 June

THE HARMONIC ANALYSIS OF THE EARTH'S MAGNETIC FIELD, FOR EPOCH 1942

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(Received 1952 May 1)

Summary

The results are given of the harmonic analysis of the Admiralty magnetic charts of declination, horizontal intensity and inclination for the epoch 1942·5. Within the limits of observational error, the Earth's magnetic field appears to be entirely of internal origin. There is no evidence of a dipole field of external origin greater than 0·1 per cent of the field of internal origin. The intensity of the dipole field is at present decreasing at a rate of about 5 per cent per century. The geomagnetic poles have a westerly drift at a rate of 4°·5 per century; the north magnetic dip pole is moving in a direction a little to the west of north, but the south magnetic dip pole appears to be practically stationary. In consequence of the dearth of magnetic data over the oceans since 1929, magnetic charts are becoming less accurate and there is a great need for airborne magnetic surveys of ocean areas.

1. In a paper entitled "The Earth's Magnetic Potential", by Dyson and Furner*, a harmonic analysis of the Earth's magnetic field was given, based on data from the *Admiralty Magnetic Charts for 1922*. These charts, for magnetic declination, horizontal intensity and inclination, were compiled at the Royal Observatory.

2. Declination charts have been prepared at five-year intervals, while charts for other components of the magnetic field have been prepared at less frequent intervals. Charts for horizontal intensity and inclination, in addition to declination, were last prepared for the epoch 1942·5. In future, in accordance with a recommendation of the Association of Terrestrial Magnetism of the International Union for Geodesy and Geophysics, charts for declination will be produced at five-year intervals, 1955, '60, '65, etc., and charts for horizontal intensity, vertical intensity, total intensity and inclination at ten-year intervals, 1955, '65, '75, etc. Preparations for the compilation at the Observatory of the complete series of charts for the epoch 1955·0 have accordingly been commenced.

* F. Dyson and H. Furner, *M.N., Geophys. Suppl.*, 1, 76, 1923.

3. Since the unfortunate loss of the non-magnetic ship, the *Carnegie*, in 1929, practically no magnetic data for the sea areas have been obtained. Both the values of the elements of the field at epoch and their secular change have consequently become increasingly uncertain over the sea areas with lapse of time. It therefore seemed desirable to make a harmonic analysis of the Earth's magnetic field, as depicted by the *Admiralty Magnetic Charts for 1942*, in the expectation that it would provide some indication of the regions in which the charted data were seriously in error and that comparison between the analysis for the epochs 1922 and 1942 would provide information about the secular change of the field between those epochs in much greater detail than could be obtained from the scanty observations alone. The analysis was completed in 1945, but pressure of work has hitherto prevented the results from being written up.

4. Values of the declination, horizontal intensity and inclination were read off from the charts at points of a grid spaced at 10° intervals in longitude and latitude between 80° N. and 80° S. latitudes. From the values so tabulated, the northerly, easterly and vertical components of the field were calculated; the values of these components are tabulated in Tables I–III.

If the magnetic potential is assumed to arise solely from forces situated inside the Earth, it can be expressed in the usual form:

$$V=a\sum_{n=0}^{\infty}\sum_{m=0}^{n}\left(\frac{a}{r}\right)^{n+1}\{H_n^m(\lambda)(g_n^m\cos m\phi+h_n^m\sin m\phi)\},$$

where a denotes the radius of the Earth, r the distance from the centre, and g_n^m and h_n^m are numerical coefficients,

$$H_n^m(\lambda)=\cos^m\lambda\left\{\mu^{n-m}-\frac{(n-m)(n-m-1)}{2(2n-1)}\mu^{n-m-2}+\frac{(n-m)(n-m-1)(n-m-2)(n-m-3)}{2.4(2n-1)(2n-3)}\mu^{n-m-4}-\dots\right\},$$

ϕ is the longitude, λ the latitude, and μ stands for $\sin\lambda$. The nomenclature due to C. F. Gauss and used by J. C. Adams (*Collected Papers*, Vol. II, "The Theory of Terrestrial Magnetism") is here followed. The symbols g and h are known as the Gauss coefficients.

From this expression, if X, Y, Z denote the northerly, easterly and vertical (downwards) components, it follows that at the surface (where $r=a$):

$$X=\frac{1}{r}\frac{\partial V}{\partial\lambda}=\Sigma\frac{dH_n^m}{d\lambda}\{g_n^m\cos m\phi+h_n^m\sin m\phi\},$$

$$Y=\frac{1}{r\cos\lambda}\frac{\partial V}{\partial\phi}=\Sigma mH_n^m\sec\lambda\{-g_n^m\sin m\phi+h_n^m\cos m\phi\},$$

$$Z=-\frac{\partial V}{\partial r}=\Sigma(n+1)H_n^m\{g_n^m\cos m\phi+h_n^m\sin m\phi\}.$$

The first six harmonics were included in the solution.

5. Though the functions H_n^m have been extensively used in harmonic analysis by Laplace, Gauss, Adams and others, Adolf Schmidt introduced new orthogonal functions, $P_n^m(\theta)$, defined by

$$P_n^m(\theta)=P_{n,m}(\theta),\qquad\text{when } m=0,$$

and

$$P_n^m(\theta)=\left\{2\frac{(n-m)!}{(n+m)!}\right\}^{1/2}P_{n,m}(\theta),\qquad\text{when } m>0,$$

TABLE I

X, Northerly Force. Unit 0·001 gauss

	+80°	70°	60°	50°	40°	30°	20°	+10°	0°	−10°	20°	30°	40°	50°	60°	70°	−80°
0° E.	+70	+106	+145	+188	+236	+281	+310	+313	+272	+225	+182	+151	+140	+146	+158	+167	+162
10	71	110	148	193	243	290	321	321	283	231	181	143	128	132	145	155	152
20	72	112	151	197	249	295	331	331	291	238	182	140	123	123	139	146	145
30	71	110	150	198	253	299	334	337	299	244	185	139	123	123	133	137	133
40	67	105	148	199	255	303	342	345	308	253	193	145	129	128	128	126	123
50	62	100	145	200	257	307	350	353	317	263	205	155	137	131	121	115	111
60	54	91	142	201	262	314	358	364	333	273	216	165	145	130	112	105	96
70	45	82	138	201	267	324	370	381	351	289	228	176	150	126	102	85	80
80	36	75	133	202	275	336	381	393	365	306	241	186	149	113	83	60	56
90	29	72	129	204	283	346	392	400	377	322	259	197	148	102	70	34	+ 26
100	27	71	129	207	288	352	394	400	385	340	279	214	152	97	60	26	0
110	26	73	132	214	291	349	389	400	387	351	297	233	163	96	53	22	− 26
120	28	76	140	220	289	342	374	392	386	357	311	251	175	100	52	20	47
130	33	81	149	225	285	331	357	376	380	360	320	262	187	109	57	16	64
140	37	94	162	229	282	317	340	361	371	359	325	270	200	119	61	16	81
150	39	106	173	231	276	306	325	345	361	356	327	275	209	133	74	10	88
160	41	114	178	232	268	293	311	332	351	353	327	280	217	144	85	35	91
170 E.	41	118	180	227	258	280	300	321	346	350	328	283	225	159	101	27	91
180	38	115	176	220	249	270	290	313	340	346	326	285	232	172	114	17	88
170 W.	33	107	167	212	240	263	285	311	338	341	325	288	240	183	126	19	78
160	28	94	154	201	234	261	284	310	332	334	322	289	246	194	134	24	77
150	21	79	140	191	228	262	288	312	327	328	319	289	251	203	142	37	33
140	+ 9	63	123	181	225	266	293	314	324	323	312	288	254	211	151	40	− 12
130	−10	46	105	168	222	269	299	318	323	319	308	285	256	218	161	70	+ 15
120	13	29	91	157	217	269	303	320	321	317	303	281	257	222	173	98	54
110	11	12	77	143	208	265	305	321	320	314	298	276	255	227	186	123	82
100	10	5	68	131	199	259	303	319	318	309	291	270	252	231	197	148	109
90	21	9	62	120	185	247	296	313	315	305	286	264	250	238	213	173	129
80	23	12	60	116	176	234	284	304	308	299	278	257	246	244	226	194	145
70	−13	18	66	119	171	222	268	294	300	290	271	249	241	245	237	209	160
60	+12	35	75	124	171	216	257	281	290	279	259	239	233	241	240	217	174
50	29	48	88	132	177	218	252	272	280	268	247	225	219	230	238	222	181
40	41	64	104	146	188	226	254	270	272	254	231	207	204	215	226	221	181
30	51	77	118	158	202	241	264	274	264	242	214	190	187	197	210	213	180
20	58	90	130	171	217	256	281	285	261	231	199	175	170	179	191	201	177
10 W.	+63	+100	+140	+181	+228	+269	+298	+300	+264	+226	+189	+162	+153	+161	+172	+183	+169

TABLE II

Y, Easterly Force. *Unit 0·001 gauss*

	+80°	70°	60°	50°	40°	30°	20°	+10°	0°	−10°	20°	30°	40°	50°	60°	70°	−80°
0° E.	−19	−26	−29	−33	−37	−42	−49	−59	−69	−77	−81	−78	−75	−71	−68	−62	−59
10	9	−14	−15	18	21	25	33	41	50	59	66	67	67	67	74	75	71
20	−1	+1	+1	−3	−6	−11	18	25	34	43	51	55	58	68	81	85	84
30	+7	15	14	+11	+7	+1	−6	13	20	31	39	45	56	73	91	98	96
40	16	25	23	20	17	10	+3	4	14	22	33	44	61	83	103	108	107
50	22	33	31	28	23	16	7	2	14	26	41	54	74	98	111	117	115
60	34	39	38	32	27	16	+4	11	26	41	58	72	100	115	119	123	119
70	40	42	39	34	24	9	−4	19	36	52	69	90	122	129	126	128	123
80	36	36	34	30	18	+4	9	20	36	52	69	88	116	125	124	129	126
90	31	28	23	19	+10	0	9	15	22	37	54	74	95	106	107	117	123
100	25	18	+11	+4	0	−3	−4	−3	−4	−14	31	51	72	85	85	96	113
110	21	+8	−5	−14	−15	9	0	+10	+12	+7	−7	25	47	60	65	78	99
120	19	−3	23	32	29	18	−3	12	21	19	+12	−3	−21	35	46	63	81
130	19	13	37	43	39	23	5	12	24	28	24	+16	+1	−11	−25	37	64
140	20	19	40	44	36	19	−1	16	30	35	35	32	23	+10	0	−25	38
150	20	15	33	34	23	−7	+9	24	37	45	47	45	41	30	+18	0	−15
160	22	−6	−19	−16	−1	+12	25	37	46	53	56	57	56	47	34	+36	+8
170 E.	24	+5	0	+6	+18	30	42	49	54	60	63	67	68	61	49	74	28
180	26	18	+22	29	36	44	50	55	60	64	68	72	75	70	64	95	51
170 W.	29	31	42	48	52	54	54	53	55	60	68	74	77	77	79	110	78
160	31	42	57	63	65	62	56	51	52	57	65	74	78	83	91	126	91
150	32	49	66	75	74	67	58	51	52	56	64	74	79	87	99	136	124
140	32	53	72	81	80	71	61	52	51	55	63	73	81	91	106	150	135
130	26	51	71	79	79	73	64	56	53	55	62	73	83	96	113	149	145
120	15	45	61	71	74	71	64	57	54	55	61	73	86	101	117	145	149
110	+4	33	42	55	61	63	60	56	55	55	62	75	91	106	121	141	142
100	0	+13	+16	30	39	48	52	52	55	58	66	78	94	110	123	133	129
90	−8	−20	−11	+2	+15	27	38	45	50	56	64	76	93	108	115	121	116
80	19	39	37	−28	−15	+1	+15	+26	35	44	52	63	77	93	102	108	102
70	36	54	56	53	43	−30	−16	−5	+4	+14	+26	38	53	69	81	89	82
60	44	59	71	72	66	60	51	41	−31	−20	−8	+5	+22	39	55	66	57
50	45	62	76	79	80	82	79	74	64	54	42	−27	−11	+7	+27	43	35
40	46	61	75	80	84	94	98	96	91	82	69	54	38	−20	−2	+19	+16
30	41	57	70	73	79	87	98	107	105	97	85	71	59	43	27	−4	0
20	35	50	60	62	67	75	86	98	102	100	91	82	71	59	46	25	−19
10 W.	−30	−39	−46	−48	−52	−60	−69	−81	−88	−93	−90	−82	−76	−67	−59	−47	−39

TABLE III

Z, Vertical Force. Unit 0·001 *gauss*

	+ 80°	70°	60°	50°	40°	30°	20°	+ 10°	0°	− 10°	20°	30°	40°	50°	60°	70°	− 80°
0° E.	+518	+501	+470	+423	+363	+262	+150	+ 18	−103	−179	−219	−238	−260	−300	−354	−428	−513
10	518	499	467	423	362	258	138	0	129	207	249	271	286	315	372	441	519
20	519	502	469	425	364	261	135	0	134	226	269	287	301	330	392	457	526
30	522	505	477	430	371	271	142	+ 5	131	233	281	294	313	345	418	473	535
40	530	511	486	445	385	284	157	17	126	228	282	300	328	371	439	489	543
50	540	520	496	464	403	296	171	29	112	217	282	313	351	409	455	505	552
60	549	542	511	483	412	310	185	40	98	205	289	328	390	441	472	522	562
70	558	555	529	501	429	325	195	40	107	214	303	348	425	459	491	536	572
80	564	566	547	514	447	340	201	34	122	234	318	367	441	474	512	548	583
90	569	576	559	530	460	348	202	30	132	250	336	395	449	482	525	558	592
100	573	586	569	544	469	349	201	32	132	272	369	429	470	480	524	568	605
110	577	595	577	548	473	340	194	39	121	270	385	465	502	475	525	577	621
120	578	603	580	547	457	325	187	43	109	253	387	491	524	479	527	589	639
130	578	602	578	539	429	306	177	47	98	236	384	495	542	506	535	610	650
140	577	593	571	510	401	281	167	48	87	227	370	491	560	542	555	647	660
150	573	584	563	475	366	260	158	49	73	213	351	484	563	566	600	690	665
160	569	574	547	442	342	249	155	57	59	199	330	468	559	578	647	685	668
170 E.	565	563	526	418	330	252	168	71	39	182	310	442	536	579	662	682	666
180	560	552	506	412	331	261	182	90	− 20	164	292	419	501	574	657	675	665
170 W.	565	551	495	417	339	273	199	105	+ 2	136	264	386	480	553	650	660	658
160	572	558	497	439	359	292	214	119	14	119	243	362	459	526	618	650	650
150	579	572	521	463	390	313	227	130	20	101	226	338	428	498	564	637	645
140	586	586	552	494	423	341	241	139	29	87	209	316	399	474	525	587	628
130	592	596	581	531	456	370	262	150	37	77	190	293	377	455	507	555	607
120	597	605	592	557	490	393	288	169	46	66	172	268	354	436	492	538	586
110	598	615	605	584	520	419	309	186	60	50	152	240	326	410	474	521	568
100	595	620	615	601	556	453	336	209	81	28	126	210	294	377	446	505	554
90	591	610	610	602	562	483	362	230	107	− 1	96	178	257	338	421	488	543
80	585	598	598	586	549	481	379	255	131	+ 30	64	142	214	293	384	467	534
70	578	584	578	566	530	472	381	266	149	50	46	115	184	266	353	448	525
60	566	569	564	544	504	445	364	265	153	59	38	104	164	237	326	431	518
50	553	552	541	508	467	407	333	249	139	42	42	106	159	227	313	418	512
40	539	534	522	484	435	371	291	201	100	+ 9	62	119	167	228	310	410	509
30	531	519	499	462	406	335	238	147	+ 50	− 40	104	144	183	236	312	408	507
20	525	510	488	441	386	302	200	95	− 10	91	143	173	208	256	323	411	507
10 W.	519	503	480	429	371	276	171	50	− 60	−139	180	204	236	276	338	417	509

where $P_{n,m}$ denotes the associated Legendre function defined by

$$P_{n,m}(\mu)=(1-\mu^2)^{m/2}\frac{d^m P_n(\mu)}{d\mu^m},$$

where $\mu=\cos\theta,$

θ being the co-latitude, i.e. $(\pi/2-\lambda)$.

It is readily shown that $P_{n,m}$ is related to H_n^m by the relation

$$P_{n,m}=\frac{(2n)!}{2^n . n!(n-m)!}H_n^m$$
$$=\frac{(2n-1)!!}{(n-m)!}H_n^m.$$

H_n^m is identical with the function which Schmidt denotes by $P^{n,m}$.

It follows that

$$P_n^m=\left\{2\frac{(n-m)!}{(n+m)!}\right\}^{1/2}P_{n,m}$$
$$=\left\{2\frac{(n-m)!}{(n+m)!}\right\}^{1/2}\frac{(2n-1)!!}{(n-m)!}P^{n,m}$$
$$=\xi P^{n,m} \qquad \text{(say)}.$$

The values of $\xi\equiv P_n^m/P^{n,m}$ for $n=1$ to 6, $m=0$ to 6 have been tabulated by Schmidt.*

The use of the functions P_n^m in geophysical investigations in preference to the functions H_n^m has been recommended by S. Chapman. Their properties are described in *Geomagnetism*, Vol. II, Chap. XVII, by Chapman and Bartels.

Schmidt (*loc. cit.*) has tabulated the values of the function $P_n^m(\cos\theta)$ and the associated functions

$$X_n^m=dP_n^m(\cos\theta)/n\,d\theta;\quad Y_n^m=mP_n^m(\cos\theta)/n\sin\theta$$

for values of m and n up to 6 and for values of θ (the co-latitude) at intervals of 5° from 0° to 90°. His tables can therefore be used to obtain the values of $dH_n^m/d\lambda$, $mH_n^m\sec\lambda$ and $(n+1)H_n^m$ in the expressions for X, Y, Z above.

6. Vestine† carried out a harmonic analysis of the U.S. magnetic charts for epoch 1945. He expresses the magnetic potential from internal forces in the form:

$$V=a\Sigma\Sigma\left(\frac{a}{r}\right)^{n+1}\frac{1}{n}P_n^m(A_n^m\cos m\phi+B_n^m\sin m\phi)$$

and, at the surface,

$$X=\Sigma X_n^m\{A_n^m\cos m\phi+B_n^m\sin m\phi\},$$
$$Y=\Sigma Y_n^m\{-A_n^m\sin m\phi+B_n^m\cos m\phi\},$$
$$Z=\Sigma\frac{n+1}{n}P_n^m\{A_n^m\cos m\phi+B_n^m\sin m\phi\},$$

where X_n^m, Y_n^m, P_n^m are as tabulated by Schmidt.

In the Gaussian notation

$$X_n^m=-\frac{\xi}{n}dH_n^m/d\lambda;\quad Y_n^m=\frac{\xi}{n}(mH_n^m\sec\lambda);\quad P_n^m=\xi H_n^m.$$

* A. Schmidt, *Tafeln der Normierten Kugelfunctionen*, Gotha, 1935, p. 20.

† E. H. Vestine and others, *The Geomagnetic Field, Its Description and Analysis*, Carnegie Institution, Washington. Publication 580, 1947.

The values of the Gaussian coefficients g_n^m, h_n^m are therefore related to the coefficients A_n^m, B_n^m used by Vestine by the formulae

$$g_n^m = -\frac{\xi}{n} A_n^m; \quad h_n^m = -\frac{\xi}{n} B_n^m.$$

These formulae can be used for comparing the results of the analysis by Vestine of the American charts for epoch 1945 with those of the present analysis of the British charts for epoch 1942.

It should be noted that Vestine, in his Table IX, tabulates the first eight Gaussian coefficients of the Earth's magnetic potential from a series of analyses going back to the earliest, by Gauss, 1835. These coefficients are g_n^m, h_n^m up to the value of 2 for m and n. The tabulated values are not, however, those of the coefficients as defined by Gauss, but those of the coefficients A_n^m, B_n^m. It is advisable that a distinction should be made in the nomenclature for, and the designation of, the coefficients according to whether the expressions used by Gauss or by Schmidt are adopted.

7. The values of X, Y, Z, given in Tables I, II and III, were analysed to give the g and h coefficients, the different latitude zones being combined together with the relative weights used by Dyson and Furner, viz.:—

Lat.	Wt.	Lat.	Wt.	Lat.	Wt.
0°	10	±30°	8	±60°	3
±10°	10	±40°	7	±70°	2
±20°	9	±50°	5	±80°	1

These preliminary solutions having been made, the weighted residuals for each latitude zone for each of the solutions were tabulated, and the mean residuals obtained for X, Y, Z separately. The mean residuals corresponding to unit weight for each latitude zone and for each component were then obtained. For X and Y the zonal values ran smoothly, but in Z the scatter was large, particularly in southern latitudes. For each component the mean residual was plotted against latitude and a smooth curve was drawn through the plotted points. The values read off from this curve were adopted as the mean residual for each zone of latitude and for each component. From these values, relative weights were obtained, which represent more accurately the uncertainties in the chart values than the provisional arbitrary weights originally used. The relative weights so derived, which were used in new solutions to obtain the definitive values of the g and h coefficients, are given in Table IV.

8. It will be seen that in general the weight for each component for a southern latitude belt is appreciably less than the weight for the corresponding northern latitude belt. This is because the land areas are more extensive in the northern hemisphere than in the southern; over many of the land areas the magnetic data are reasonably well known from land magnetic surveys, whereas there have been very few magnetic observations in the sea areas since 1929. An additional reason is that there are many more permanent magnetic observatories and secular change repeat stations in the northern hemisphere than in the southern, in consequence of which the secular change data are more reliable for the northern hemisphere than for the southern.

The weights in Table IV have been formed on a uniform basis for each component. It will be noted that the weights in X and Y are of the same order, the weights in Y being mostly greater than those in X in northern latitudes, but somewhat smaller in southern latitudes. The X and Y values are formed from the observed values of declination and of horizontal intensity; for most of the region of the Earth's surface between 60° N. and 60° S. latitude, the declination does not exceed 30°. A given error in H will therefore produce a larger error in X than in Y, while a given error in D will produce a larger error in Y than in X. In the southern Indian Ocean the declination is known to be very uncertain, as in this region the secular change in declination is large, while the rate of change of declination with latitude is also large.

TABLE IV

Relative Weights for each Latitude Belt and for each Component (X, Y, Z)

Lat.	X	Y	Z
+80°	1·61	2·06	0·35
70	2·63	3·05	0·46
60	4·35	5·00	0·63
50	6·95	7·58	0·87
40	8·62	11·40	1·25
30	10·2	15·2	1·51
20	11·9	19·2	1·54
+10	13·2	20·8	1·39
0	13·9	19·2	1·24
−10	13·5	15·2	1·01
20	11·1	10·9	0·76
30	8·20	7·25	0·53
40	6·17	5·00	0·35
50	3·79	3·47	0·18
60	2·28	1·79	0·15
70	0·74	0·80	0·13
−80	0·36	0·42	0·13

The weights of the vertical intensity are relatively low. This component is formed from the charted values of horizontal intensity and inclination. It seems probable that there are appreciable errors in the inclination: over much of the surface of the globe, moreover, the inclination exceeds 45°, so that the errors in inclination are much enhanced in the vertical intensity.

9. If a portion of the Earth's magnetic field is of external origin, the magnetic potential will be expressible in the form

$$V = a\Sigma \left(\frac{a}{r}\right)^{n+1} [H_n{}^m(g_n{}^m \cos m\phi + h_n{}^m \sin m\phi)],$$
$$+ a\Sigma \left(\frac{r}{a}\right)^n [H_n{}^m (g_{-n}{}^m \cos m\phi + h_{-n}{}^m \sin m\phi)].$$

The analysis, in the form explained above, by which the coefficients of the terms depending on the longitude are derived will determine

$$\text{for } X, Y \qquad g_n{}^m + g_{-n}{}^m,$$
$$\text{for } Z \qquad g_n{}^m - \frac{n}{n+1} g_{-n}{}^m.$$

The coefficients determined from the separate analyses of the northerly and easterly components of the magnetic field should therefore agree, apart from the effect of accidental errors. The values of the coefficients obtained from the analysis of these two components, with the weights first assumed, were in fact in close agreement. A new solution was accordingly made by combining the data for these two components and using the definitive weights given in Table IV. The derived values of the coefficients are given in the second column of Table V. The vertical component was analysed separately, using the definitive weights given in Table IV, and the derived values of the coefficients are given in the third column of Table V.

10. If there is an external component of the total field the difference between the values of g_n^m derived separately from $(X+Y)$ and Z is given by $(2n+1)g_{-n}^m/(n+1)$. In comparing the values given in columns 2 and 3 of Table V it should be noted that the values in column 2 have a weight that is seven or eight times the weight of the values in column 3, the combined weight being given in column 5, and that the probable error corresponding to unit weight is about $\pm 0{\cdot}0035$. From an examination of columns 2 and 3, in conjunction with the relative weights and probable errors, no evidence of the existence of an external component of the field is found. In relation to the probable errors of the various quantities, there are no significant differences between the corresponding values in columns 2 and 3.

The main portion of the field is represented by the dipole field of a uniformly magnetized sphere (Section 16). The intensity of this field, H_0, the co-latitude, θ_0, and the longitude, ϕ_0, of the northern pole of this field, as derived separately from the north and east and from the vertical components of the field, are as follows :—

	H_0	θ_0	ϕ_0
From X and Y	$0{\cdot}3096 \pm 0{\cdot}0004$	$11^\circ{\cdot}1$	$-68^\circ{\cdot}4$
From Z	$0{\cdot}3104 \pm 0{\cdot}0010$	$11^\circ{\cdot}0$	$-69^\circ{\cdot}4$

It can be concluded from these results that there is no evidence of a dipole field of external origin which exceeds one-tenth of one per cent of the field of internal origin.

It has therefore been assumed that there is no portion of the Earth's magnetic field that is of external origin. The equations for the $(X+Y)$ and the Z components were therefore combined together with appropriate weights and solved to give the values listed in column 4 of Table V, the relative weights being given in column 5.

11. It is of some interest to compare the results of the present analysis with the results obtained by Vestine. The charts from which the data for the two analyses were read off differ in epoch by only three years. Though the British and American charts were prepared quite independently, they are based essentially on the same data. But over many areas recent observations are completely lacking and both series of charts consequently have been based in some areas on extrapolation, involving the assessment of the secular change of the Earth's magnetic field and on the rate of change of this secular change.

The comparison between the Gaussian coefficients and the first four harmonics from $(X+Y)$, and from Z, as derived by Vestine and in the present investigation, is given in Table VI.

The agreement between the coefficients derived from the north and east components is satisfactory. The discordances between those derived from the vertical component are appreciably greater, which is to be expected from the

TABLE V

Values of Coefficients

	From X and Y	From Z	From X, Y and Z (spherical Earth)	Relative weights	From X, Y and Z (spheroidal Earth)
g_1^0	+·303 8	+·304 7	+·3039	97	+·3022
g_2^0	+·017 8	+·015 9	+·0176	59	+·0176
g_3^0	−·025 5	−·024 5	−·0255	28	−·0292
g_4^0	−·039 7	−·039 7	−·0398	12	−·0385
g_5^0	+·030 3	+·021 1	+·0293	5	+·0182
g_6^0	−·021 0	−·021 9	−·0211	2	−·0142
g_1^1	+·021 9	+·020 8	+·0218	193	+·0218
g_2^1	−·051 2	−·047 6	−·0509	94	−·0507
g_3^1	+·052 4	+·044 3	+·0515	38	+·0508
g_4^1	−·040 5	−·032 9	−·0397	15	−·0384
g_5^1	−·030 2	−·042 6	−·0329	6	−·0342
g_6^1	−·007 5	−·011 7	−·0073	2	−·0054
h_1^1	−·055 4	−·055 4	−·0555	193	−·0553
h_2^1	+·026 6	+·022 4	+·0260	94	+·0259
h_3^1	+·019 4	+·016 2	+·0190	38	+·0194
h_4^1	−·014 1	−·010 1	−·0139	15	−·0143
h_5^1	+·006 1	−·001 2	+·0057	6	+·0053
h_6^1	−·002 3	−·003 0	−·0026	2	−·0019
g_2^2	−·013 3	−·015 9	−·0135	532	−·0135
g_3^2	−·023 9	−·019 4	−·0236	115	−·0235
g_4^2	−·023 6	−·027 9	−·0238	32	−·0234
g_5^2	−·013 5	−·007 3	−·0130	10	−·0123
g_6^2	−·001 0	+·000 5	−·0007	3	+·0002
h_2^2	−·004 2	−·006 8	−·0044	532	−·0044
h_3^2	−·003 4	−·000 4	−·0033	115	−·0033
h_4^2	+·008 5	−·001 6	+·0076	32	+·0076
h_5^2	−·002 4	+·007 0	−·0018	10	−·0017
h_6^2	−·018 9	−·036 5	−·0204	3	−·0206
g_3^3	−·007 5	−·006 6	−·0074	983	−·0074
g_4^3	+·008 9	+·005 3	+·0087	139	+·0087
g_5^3	+·002 3	+·012 9	+·0031	31	+·0033
g_6^3	+·021 7	+·011 1	+·0210	8	+·0205
h_3^3	·000 0	−·001 7	−·0001	983	−·0001
h_4^3	+·001 6	+·005 0	+·0019	139	+·0019
h_5^3	+·000 7	+·000 7	+·0009	31	+·0009
h_6^3	+·000 8	+·010 0	+·0018	8	+·0017
g_4^4	−·002 0	+·000 1	−·0018	1523	−·0018
g_5^4	+·003 3	+·003 9	+·0034	168	+·0033
g_6^4	+·002 1	−·003 8	+·0017	30	+·0018
h_4^4	+·001 0	+·000 9	+·0010	1523	+·0010
h_5^4	+·003 4	+·001 9	+·0032	168	+·0032
h_6^4	+·000 8	+·003 1	+·0009	30	+·0009
g_5^5	+·000 5	+·000 2	+·0005	2189	+·0005
g_6^5	−·000 4	·000 0	−·0004	196	−·0004
h_5^5	−·000 5	−·000 4	−·0004	2189	−·0004
h_6^5	+·000 5	−·000 6	+·0004	196	+·0005
g_6^6	+·000 6	+·000 3	+·0006	2835	+·0006
h_6^6	+·000 2	+·000 2	+·0002	2835	+·0002

relatively low weight of Z from the British charts, as shown by Table IV. The agreement between the coefficients derived from the elements $(X+Y)$ and Z is better from the American charts than from the British; the American charts of vertical intensity are the better.

TABLE VI

Gaussian coefficients derived by Vestine and by Jones and Melotte

	$X+Y$		Difference	Z		Difference
	Vestine	Jones and Melotte		Vestine	Jones and Melotte	
g_1^0	+·305 7	+·303 8	+·0019	+·305 7	+·304 7	+·0010
g_2^0	+·019 0	+·017 8	+ 12	+·017 9	+·015 9	+ 20
g_3^0	−·028 7	−·025 5	− 32	−·026 7	−·024 5	− 22
g_4^0	−·040 3	−·039 7	− 6	−·043 7	−·039 7	− 40
g_1^1	+·021 0	+·021 9	− 9	+·022 7	+·020 8	+ 19
g_2^1	−·051 2	−·051 2	0	−·051 0	−·047 6	− 34
g_3^1	+·051 9	+·052 4	− 5	+·053 8	+·044 3	+ 95
g_4^1	−·043 0	−·040 5	− 25	−·041 5	−·032 9	− 86
h_1^1	−·058 1	−·055 4	− 27	−·057 9	−·055 4	− 25
h_2^1	+·028 7	+·026 6	+ 21	+·029 7	+·022 4	+ 73
h_3^1	+·015 9	+·019 4	− 35	+·014 3	+·016 2	− 19
h_4^1	−·007 8	−·014 1	+ 63	−·006 4	−·010 1	+ 37
g_2^2	−·014 1	−·013 3	− 8	−·014 4	−·015 9	+ 15
g_3^2	−·023 4	−·023 9	+ 5	−·023 3	−·019 4	− 39
g_4^2	−·022 5	−·023 6	+ 11	−·023 2	−·027 9	+ 47
h_2^2	−·004 6	−·004 2	− 4	−·004 1	−·006 8	+ 27
h_3^2	−·003 5	−·003 4	− 1	−·004 0	−·000 4	− 36
h_4^2	+·010 8	+·008 5	+ 23	+·011 4	−·001 6	+ 130
g_3^3	−·006 9	−·007 5	+ 6	−·007 3	−·006 6	− 7
g_4^3	+·007 9	+·008 9	− 10	+·008 5	+·005 3	+ 32
h_3^3	−·000 4	·000 0	− 4	·000 0	−·001 7	+ 17
h_4^3	+·001 7	+·001 6	+ 1	+·002 2	+·005 0	− 28
g_4^4	−·002 2	−·002 0	− 2	−·002 9	+·000 1	− 30
h_4^4	+·000 9	+·001 0	− 1	+·000 9	+·000 9	0

Vestine has not given the weights of his coefficients; he used the weights that were adopted by Dyson and Furner for combining the data for different latitudes, which give the southern latitudes too great a weight relative to the northern. On the basis of the assumption, which is not strictly correct, that the weights of each coefficient deduced from the X and Y charts are equal in the two investigations, it was found, from a comparison of all the coefficients for the first six harmonics, that the probable error corresponding to unit weight was $\pm 0{\cdot}0054$. The value obtained in the present investigation was $\pm 0{\cdot}0035$. The concordance of these two values is sufficiently close, in view of the fact that the basic assumption about the weights is not strictly accurate, to suggest that the weights of the two determinations are not greatly different and that the discordances between the individual coefficients are mainly accidental.

12. With the adopted values of the coefficients, given in column 4 of Table V, the values of X, Y, Z at each point of the 10° network were computed and compared with the values derived from the charts. The differences between the chart data and the computed values are given for the three elements respectively in Tables VII, VIII and IX. The quantities in these tables necessarily run on the whole smoothly, because the chart data are smoothed to eliminate as far as possible the effects of local magnetic anomalies, while the computed values necessarily run smoothly. Comparison between these tables and the corresponding tables in Dyson and Furner's paper (*loc. cit.* pp. 85–87) is of interest. In Tables VII and VIII there are several regions in which the residuals are systematic and of appreciable magnitude. The residuals in these tables are larger on the average than those based on the 1922 charts, an indication that the 1942 charts are of lower accuracy than the 1922 charts, as would be expected from the paucity of ocean data since 1929. There is, moreover, little correlation between the corresponding residuals from the 1922 and 1942 charts; if the charts were of high accuracy and the same regions stood out in the same direction at different epochs, it would indicate that the general magnetic field was not represented with adequate accuracy by the six harmonics. The comparison between the 1922 and 1942 residuals suggests that there must be appreciable errors in the secular change field that was adopted for bringing forward observations to the epoch 1942·5 of the later charts, while the large systematic runs of residuals suggest that there are areas where the charts are appreciably in error.

Comparison between the tables clearly shows that the vertical force data are of much lower reliability than those for the north and east components. The largest residuals are found in regions where the errors in horizontal intensity and in inclination are additive and where, in addition, the tangent of the dip has a large value. In middle latitudes (between 50° N. and 50° S.) the residuals tend to be larger than those obtained from the 1922 charts, but in the polar caps the very large residuals obtained from the 1922 charts have been reduced.

13. The charts on which this analysis was based were charts of declination, horizontal intensity and inclination. From the values of X, Y and Z computed from the coefficients listed in Table V, the values of these three components were derived. Tables X, XI, XII give the differences between the chart values and the computed values for the three elements declination, horizontal intensity and inclination. Table X is of special interest because of the use of the charts of declination for navigational purposes. Too much regard need not be paid to the large residuals in high north and south latitudes, because of the proximity to the magnetic and geographical poles, which are singular points in the system of isogonals. In latitudes 50° S. and 60° S. there are some considerable residuals in the southern Indian Ocean, which suggest that in places in this area the charts may be at least 5° in error. The southern Indian Ocean has been recognized for many years as a difficult region, for not only are the isogonals more closely spaced there than elsewhere, but in addition it is a region where the secular change of declination is both large and ill-determined. It is clear from Table X that the charts in declination are not of the accuracy that is needed for certain modern developments in navigational aids.

TABLE VII

X, Northerly Force. Chart minus Computed. Unit 0·001 *gauss*

	+80°	70°	60°	50°	40°	30°	20°	+10°	0°	−10°	20°	30°	40°	50°	60°	70°	−80°
0° E.	−6	−6	−1	0	0	+1	+3	+7	−6	−6	0	+5	+4	−5	−19	−25	−15
10	−8	−6	−2	+2	+1	0	+2	+3	−3	−3	+2	+3	0	−10	−22	−25	−13
20	−8	−5	0	+3	+3	−2	+2	+3	−4	0	+2	+2	−2	−14	−20	−23	−6
30	−7	−5	−1	+2	+4	−2	−1	+1	−4	−1	0	−2	−3	−13	−20	−21	−2
40	−7	−5	−2	+2	+3	−2	+2	+1	−4	−1	0	−3	−1	−8	−20	−19	+6
50	−6	−4	−1	+3	+2	−2	+4	+1	−5	−4	0	−2	+1	−5	−19	−15	+15
60	−6	−5	+1	+4	+3	−2	+4	+2	−3	−8	−2	−2	+5	−3	−18	−9	+21
70	−8	−5	+2	+3	+2	−1	+4	+5	0	−8	−4	0	+9	0	−15	−10	+27
80	−9	−4	+2	+3	+4	0	+2	+3	−1	−7	−6	+1	+7	−5	−18	−15	+26
90	−10	−1	0	+3	+5	+1	+4	−1	−3	−7	−3	+2	+5	−8	−15	−21	+18
100	−8	+1	0	+3	+6	+3	+3	−5	−3	−3	0	+5	+4	−7	−12	−11	−12
110	−7	+1	−1	+5	+6	+3	+3	−1	−3	−3	+2	+8	+5	−8	−10	0	+4
120	−5	−1	−1	+5	+4	+3	0	+2	−1	−3	+2	+10	+6	−8	−6	+9	−3
130	−2	−5	−3	+3	+2	+2	−1	0	0	−2	+1	+7	+5	−6	−2	+11	−9
140	−1	−2	−2	0	+2	+1	−2	0	0	−2	+1	+6	+7	−6	−3	+12	−19
150	−2	0	−2	−2	+2	+4	0	−2	−1	−2	+1	+5	+7	−3	+1	+3	−24
160	−3	0	−4	−2	+2	+5	+2	−3	−3	−1	+1	+5	+5	−4	+1	+21	−28
170 E.	−4	−1	−5	−4	+2	+6	+4	−4	−2	−1	+2	+4	+5	−2	+5	+4	−33
180	−5	−3	−6	−4	+3	+7	+3	−5	−3	−2	0	+2	+3	−1	+5	−16	−39
170 W.	−7	−6	−8	−3	+2	+7	+3	−2	0	−2	+1	+2	+2	−1	+5	−26	−40
160	−6	−9	−9	−4	+2	+6	+2	−1	0	−3	+2	+2	+2	0	+2	−34	−53
150	−5	−10	−7	−2	−1	+4	+2	+1	−1	−2	+3	+2	+2	+1	−1	−34	−26
140	−8	−10	−6	0	−1	+4	+1	+1	−1	−1	+1	+2	+3	+3	−2	−46	−23
130	−19	−10	−6	0	+1	+3	+1	+2	0	0	+3	+2	+4	+5	−3	−32	−17
120	−14	−12	−2	+3	+2	+2	0	0	−2	+2	+4	+3	+6	+5	−2	−22	0
110	−7	−17	0	+3	+1	+1	+2	−1	−3	+3	+5	+4	+6	+5	−1	−16	+5
100	−3	−16	+1	+2	+3	+3	+4	−1	−4	+1	+4	+4	+6	+4	−3	−11	+9
90	−15	−10	+1	0	0	+2	+6	−2	−3	0	+4	+4	+6	+6	−1	−5	+6
80	−21	−10	−1	+1	+1	+2	+6	−1	−4	0	+2	+3	+4	+7	0	−2	+2
70	−18	−11	−1	+3	+2	+1	+4	+1	−3	−2	+2	+2	+3	+6	+2	−2	−1
60	−3	−6	−3	+2	+2	+1	+2	−1	−2	−2	+1	+2	+3	+5	+1	−4	−2
50	+2	−8	−4	−2	+1	+1	0	−3	−1	0	+3	+2	+1	+2	+1	−5	−5
40	+2	−6	−3	−1	−1	0	−2	−2	0	−1	+2	−1	0	−1	−5	−5	−11
30	0	−7	−2	−3	−1	+1	−2	−2	−3	−1	+2	+2	+4	−2	−9	−9	−14
20	−3	−6	−2	−2	0	+1	+2	+2	−6	−4	+1	+4	+5	−2	−14	−12	−15
10 W.	−7	−6	0	0	0	+1	+5	+6	−7	−5	+1	+5	+4	−4	−19	−20	−17

TABLE VIII

Y, Easterly Force. Chart minus Computed. Unit 0·001 gauss

	+80°	70°	60°	50°	40°	30°	20°	+10°	0°	−10°	20°	30°	40°	50°	60°	70°	−80°
0° E.	+ 6	+ 1	0	−4	−5	−4	0	0	0	−2	−4	−3	− 3	− 2	− 3	− 1	− 1
10	+ 5	− 1	−1	−3	−3	−1	−1	+1	+1	−1	−4	−3	− 1	+ 1	− 2	0	+ 6
20	+ 1	0	0	−1	−1	−1	−2	−1	−1	−2	−4	−3	+ 1	− 1	− 4	+ 2	+ 9
30	− 2	0	0	0	+1	−1	−3	−3	−3	−5	−4	0	+ 1	− 3	− 6	0	+12
40	− 2	− 2	−4	−2	0	−1	−3	−3	−5	−3	−1	+2	0	− 5	− 9	+ 1	+13
50	− 3	− 2	−5	−3	−1	0	−1	−2	−3	−2	−2	+2	− 1	− 8	− 6	+ 2	+13
60	+ 4	− 2	−4	−5	0	0	−1	−4	−6	−5	−5	0	−11	−11	− 2	+ 4	+15
70	+ 8	0	−4	−3	−2	−4	−3	−5	−7	−6	−4	−6	−21	−14	− 2	+ 3	+12
80	+ 5	− 2	−3	−1	−2	−4	−4	−3	−6	−5	−3	−2	−12	− 7	+ 2	+ 1	+ 7
90	+ 3	− 1	−3	−1	0	−2	−4	−3	−1	−2	−1	+1	0	+ 4	+12	+ 6	+ 2
100	+ 1	0	0	+1	+3	+1	0	0	+1	0	0	+3	+ 4	+ 8	+19	+14	0
110	+ 3	+ 3	+1	+1	+2	+4	+4	+5	+3	+1	0	+3	+ 3	+ 9	+18	+14	− 2
120	+ 6	+ 3	−1	−1	+1	+3	+3	+3	+3	0	+1	+1	+ 2	+ 7	+12	+ 8	− 3
130	+10	+ 2	−5	−2	−2	+2	+2	+2	+2	+2	0	+2	0	+ 4	+ 7	+10	− 7
140	+13	− 1	−4	−2	0	+3	+2	+2	+4	+4	+2	+2	+ 2	+ 1	+ 7	− 2	− 4
150	+13	+ 1	−2	0	+3	+3	+1	+2	+4	+6	+5	+2	+ 2	0	+ 2	0	− 4
160	+13	+ 3	0	+3	+7	+5	+3	+3	+3	+5	+3	+2	+ 3	0	− 1	+14	− 4
170 E.	+11	+ 3	+1	+3	+5	+5	+6	+5	+3	+4	+2	+3	+ 4	+ 1	− 2	+33	− 7
180	+ 9	+ 2	+3	+4	+4	+5	+5	+6	+6	+4	+2	+2	+ 4	+ 1	0	+36	− 5
170 W.	+ 7	+ 1	+3	+2	+2	+4	+5	+4	+2	+1	+1	+2	+ 1	+ 2	+ 4	+36	+ 3
160	+ 6	+ 1	+2	0	+2	+5	+5	+4	+2	0	0	+2	0	+ 2	+ 7	+38	− 2
150	+ 6	0	0	0	+1	+3	+5	+4	+4	+2	+1	+2	− 1	+ 1	+ 6	+35	+16
140	+ 7	+ 2	+1	+1	+2	+3	+4	+3	+3	+2	+1	+1	− 1	− 1	+ 3	+37	+14
130	+ 6	+ 4	+3	+1	+1	+3	+4	+4	+3	+1	0	0	− 3	− 3	+ 2	+26	+15
120	+ 2	+ 8	+4	+2	+2	+3	+3	+2	+2	0	−2	−2	− 4	− 5	− 2	+15	+13
110	+ 1	+11	+3	+4	+3	+3	+1	0	0	−3	−3	−3	− 3	− 5	− 3	+ 8	+ 5
100	+ 8	+ 8	−1	+2	+1	+2	+1	−2	0	−1	0	−1	− 1	− 2	− 1	+ 1	− 5
90	+12	− 5	−4	0	+2	+1	+1	0	0	0	+1	+1	+ 3	+ 2	− 3	− 4	−10
80	+13	− 6	−5	−2	0	+2	+2	0	0	+1	+1	0	0	+ 1	− 2	− 4	−12
70	+ 5	− 5	−4	−3	0	+1	+1	−1	−3	−4	−2	−2	− 1	− 1	− 2	− 4	−16
60	+ 4	+ 1	−4	−3	+1	0	0	−1	−2	−3	−3	−4	− 2	− 2	− 2	− 4	−21
50	+ 7	+ 4	−1	+1	+2	0	0	−1	0	−2	−3	−2	− 3	− 3	− 2	− 2	−21
40	+ 7	+ 6	0	+1	+2	−3	−4	−2	−2	−2	−1	0	− 1	− 1	− 3	0	−17
30	+ 9	+ 5	−2	0	0	0	−3	−8	−5	−2	+1	+3	0	− 1	− 4	+ 1	− 9
20	+ 9	+ 3	−3	−2	−2	−2	−2	−6	−5	−4	+1	+2	+ 1	− 1	− 3	+ 2	− 4
10 W.	+ 5	+ 2	−3	−3	−4	−4	−3	−4	−3	−5	−3	0	0	0	− 3	− 1	− 2

TABLE IX

Z, Vertical Force. Chart minus Computed. Unit 0·001 *gauss*

	+80°	70°	60°	50°	40°	30°	20°	+10°	0°	−10°	20°	30°	40°	50°	60°	70°	−80°
0° E.	−18	+3	+9	+5	+8	0	+6	0	−6	+4	+15	+17	+4	−14	−16	−6	+10
10	−18	+2	+6	+4	+6	−3	−3	−3	+9	+3	+10	+6	−3	−15	−23	−12	+7
20	−18	+1	+4	+2	+3	−3	−2	+4	−2	−1	+6	+4	−6	−18	−31	−18	+5
30	−18	−2	+4	−1	+3	−1	−1	+5	+1	−4	0	+4	−9	−22	−45	−22	+3
40	−15	−5	+1	+1	+5	−1	−2	+6	−3	−4	−1	+3	−15	−34	−50	−24	+4
50	−10	−9	−6	+2	+8	−2	+1	+4	+1	+2	+1	−1	−23	−53	−46	−23	+6
60	−7	0	−10	+2	−2	−4	+3	+7	+12	+19	+8	+6	−35	−57	−37	−20	+6
70	−5	−2	−13	−3	−5	−2	+7	+8	+12	+28	+22	+21	−32	−37	−24	−11	+8
80	−5	−5	−15	−12	−5	+3	+11	+9	+13	+34	+42	+46	−1	−8	−10	+2	+9
90	−5	−7	−20	−13	−4	+5	+13	+12	+16	+40	+58	+61	+40	+29	+14	+17	+13
100	−6	−6	−20	−7	0	+5	+12	+14	+18	+26	+45	+60	+60	+73	+50	+31	+12
110	−5	−1	−14	−1	+10	+1	+6	+14	+19	+21	+33	+42	+58	+111	+79	+43	+7
120	−6	+8	−4	+11	+11	0	+3	+10	+13	+21	+22	+21	+52	+130	+100	+49	−1
130	−6	+11	+8	+27	+9	+2	+1	+8	+8	+19	+10	+11	+38	+115	+108	+43	−4
140	−7	+10	+20	+26	+12	+1	+2	+6	+7	+11	+9	+5	+16	+82	+96	+15	−7
150	−10	+10	+33	+20	+6	+1	+1	+3	+9	+10	+12	−4	+2	+53	+53	−22	−7
160	−13	+8	+35	+10	+3	+1	−3	+1	+8	+7	+14	−7	−10	+30	+3	−15	−8
170 E.	−16	+4	+26	0	0	+4	0	−1	+8	+3	+11	−4	−9	+15	−19	−13	−5
180	−20	−4	+11	−2	−1	+3	−1	−2	+5	−3	+3	−8	+3	+3	−24	−10	−5
170 W.	−15	−5	−3	−5	−5	0	0	−3	+9	+3	+6	−1	+1	+7	−28	−1	−1
160	−8	−2	−11	+1	−5	+1	+1	−1	+8	+2	+4	−2	0	+17	−9	0	+2
150	−2	+6	−4	0	−1	+1	+1	+3	+5	+6	+1	0	+10	+27	+30	+1	+1
140	+4	+12	+8	+2	+2	+5	+1	+4	+7	+7	+1	+1	+18	+31	+52	+37	+9
130	+10	+13	+16	+9	+2	+6	+4	+4	+7	+6	+3	+4	+17	+27	+48	+51	+20
120	+15	+15	+9	+7	+4	0	+8	+8	+4	+5	+4	+7	+14	+18	+37	+48	+30
110	+17	+21	+9	+12	+7	−1	+5	+6	+3	+6	+6	+11	+12	+11	+24	+42	+36
100	+16	+25	+12	+16	+23	+9	+7	+6	+3	+9	+10	+13	+9	+6	+18	+33	+37
90	+15	+19	+10	+14	+21	+22	+9	+1	+3	+10	+12	+12	+8	+5	+7	+24	+35
80	+13	+14	+8	+7	+11	+14	+10	+2	0	+13	+15	+15	+13	+11	+10	+20	+32
70	+11	+12	+4	+6	+8	+13	+9	+1	0	+12	+9	+14	+11	+4	+9	+16	+29
60	+5	+10	+11	+11	+10	+10	+8	+7	+4	+15	+7	+9	+9	+7	+12	+14	+26
50	−3	+8	+11	+5	+7	+8	+12	+21	+14	+15	+11	+8	+6	+4	+9	+12	+23
40	−11	+4	+14	+11	+11	+14	+16	+21	+18	+17	+17	+11	+5	+1	+4	+10	+19
30	−14	+1	+10	+13	+13	+17	+11	+19	+21	+15	+13	+14	+7	+1	+1	+7	+17
20	−15	+2	+13	+9	+15	+14	+13	+15	+11	+13	+17	+19	+7	−5	−5	+4	+14
10 W.	−18	+2	+14	+7	+12	+6	+11	+7	+4	+8	+19	+21	+5	−7	−11	0	+12

TABLE X

D, Declination (+ve. if East; −ve. if West). Chart minus Computed. Unit 0°·1

	+80°	70°	60°	50°	40°	30°	20°	+10°	0°	−10°	20°	30°	40°	50°	60°	70°	−80°
0° E.	+30	0	−4	−10	−10	−7	−1	+3	−4	−9	−13	−1	0	−16	−32	−27	−19
10	+22	−5	−4	−8	−6	−3	−3	+2	+1	−5	−10	−5	−5	−13	−38	−31	−1
20	+5	−3	0	−3	−2	−2	−2	−1	−3	−6	−10	−7	+2	−27	−42	−28	+17
30	−3	+1	−1	+2	+1	−1	−4	−7	−6	−12	−10	−2	−4	−35	−55	−35	+26
40	−7	−2	−10	−7	0	−2	−4	−5	−9	−6	−2	+3	−1	−32	−61	−35	+47
50	−15	−7	−18	−10	−3	0	−2	−4	−6	−6	−3	+3	+2	−31	−58	−31	+70
60	+55	0	−15	−15	−2	+3	−2	−6	−10	−12	−13	−5	−22	−33	−48	−15	+97
70	+107	+15	−15	−10	−5	−7	−6	−6	−10	−13	−14	−16	−36	−35	−42	−24	+117
80	+102	+6	−13	−4	−4	−8	−7	−5	−9	−12	−9	−1	−17	−29	−48	−49	+112
90	+101	−3	−12	−2	−2	−4	−5	−4	−2	−5	−3	+6	+10	−10	−26	−80	+83
100	+87	+3	+1	+3	+5	+2	+1	0	+2	0	0	+10	+14	+5	+10	−33	+63
110	+102	+23	+5	+3	+4	+6	+7	+6	+4	+2	0	+10	+16	+18	+27	+28	+21
120	+124	+25	−7	−1	+2	+5	+6	+4	+4	+1	0	+4	+10	+21	+35	+90	−4
130	+157	+4	−19	−5	−3	+4	+4	+2	+3	+2	0	+2	0	+15	+46	+216	−10
140	+181	−5	−17	−5	+1	+4	+3	+3	+6	+6	+4	+3	+4	+10	+63	+219	−41
150	+179	+7	−7	0	+6	+6	+3	+3	+6	+9	+7	+4	+4	+6	+19	−39	+3
160	+167	+16	−3	+7	+16	+10	+4	+4	+6	+8	+6	+2	+3	+7	−10	−120	+61
170 E.	+142	+14	+4	+10	+11	+9	+9	+8	+6	+6	+3	+3	+5	+7	−20	+87	+140
180	+135	+10	+14	+11	+8	+7	+9	+12	+10	+6	+4	+3	+5	+4	−10	+196	+186
170 W.	+134	+14	+7	+9	+5	+6	+9	+7	+4	+1	+2	+2	+2	+6	+3	+213	+182
160	+115	+15	+17	+4	+4	+7	+10	+7	+5	+2	0	+2	0	+5	+14	+221	+255
150	+110	+29	+10	+3	+3	+5	+8	+8	+7	+3	+1	+3	−1	0	+18	+201	+111
140	+197	+52	+15	+2	+4	+4	+7	+6	+6	+3	+1	+2	−4	−4	+11	+223	+104
130	+427	+82	+26	+3	+1	+4	+7	+5	+5	+2	−1	−2	−8	−10	+7	+147	+78
120	+448	+146	+22	+5	+3	+5	+5	+4	+3	−1	−5	−7	−12	−15	−3	+86	+18
110	+160	+318	+19	+9	+7	+5	+2	0	+1	−6	−8	−9	−13	−16	−5	+52	−6
100	−495	+573	−10	+5	−1	+5	0	−3	−1	−2	−3	−5	−6	−8	+2	+23	−32
90	−529	−267	−28	+3	+4	+3	+1	0	+1	0	0	0	0	−1	−4	0	−38
80	−464	−158	−44	−8	−1	+5	+3	+4	0	+3	+1	−2	−2	−3	−6	−6	−36
70	−275	−130	−24	−5	+3	+3	+2	0	−5	−6	−5	−5	−6	−6	−6	−8	−43
60	−25	−40	−27	−6	+6	+1	+1	−1	−4	−7	−6	−9	−7	−7	−7	−6	−60
50	+59	−25	−16	−2	+6	0	0	−2	0	−3	−4	−5	−8	−7	−5	−2	−58
40	+56	−3	−9	0	+3	−6	−8	−5	−2	−4	−1	−1	−3	−4	−8	+1	−46
30	+54	0	−10	−3	0	+1	−8	−16	−11	−6	+3	+10	+4	−2	−12	+4	−26
20	+45	−4	−17	−8	−6	−3	−3	−11	−15	−1	+2	+10	+9	−5	−17	+2	−16
10 W.	+17	0	−13	−9	−10	−8	−2	−6	−11	−15	−7	+8	+6	−7	−24	−18	−17

TABLE XI

H, Horizontal Force. Chart minus Computed. Unit 0·001 *gauss*

	+80°	70°	60°	50°	40°	30°	20°	+10°	0°	−10°	20°	30°	40°	50°	60°	70°	−80°
0° E.	−8	−7	−1	+1	+1	+1	+3	+6	−5	−5	+1	+6	+4	−4	−17	−23	−14
10	−9	−6	−1	+2	+1	0	+2	+3	−4	+1	−1	−3	+1	−9	−19	−23	−14
20	−8	−5	0	+3	+3	−2	+2	+3	−4	0	+3	+4	−2	−12	−16	−21	−11
30	−8	−5	−1	+2	+3	−2	−1	0	−3	0	+1	−2	−3	−10	−14	−18	−9
40	−7	−5	−2	+2	+4	−2	+1	+1	−4	−1	0	−4	−1	−5	−11	−16	−4
50	−6	−5	−2	+2	+2	−2	+4	+1	−5	−4	0	−3	+2	+1	−12	−13	−1
60	−3	−5	0	+3	+3	−2	+4	+2	−2	−7	−1	−2	+11	+4	−12	−8	0
70	−2	−4	0	+3	+2	−1	+4	+5	+1	−6	−3	+3	+19	+9	−8	−8	+1
80	−4	−5	0	+3	+4	0	+2	+2	0	−7	−4	+2	+13	+2	−12	−8	+2
90	−6	−2	0	+3	+5	+1	+4	−1	−2	−7	−3	+1	+4	−8	−18	−12	+1
100	−5	0	0	+3	+6	+3	+3	−5	−3	−3	+1	+4	+1	−11	−23	−17	−1
110	−3	+1	−1	+5	+6	+2	+3	−1	−4	−3	+2	+7	+5	−12	−20	−14	0
120	−1	−1	−1	+5	+3	+2	0	+2	−2	−4	+1	+10	+5	−10	−12	−6	+4
130	+2	−5	−1	+3	+2	+2	−1	0	+1	−2	+1	+8	+5	−6	−5	−7	+11
140	+3	−2	−1	0	+1	+1	−2	0	+1	−1	+1	+6	+7	−6	−4	+7	+18
150	+2	0	−1	−2	+1	+4	0	−1	0	−1	+1	+6	+7	−3	+2	+3	+24
160	+1	−1	−4	−3	+2	+5	+2	−2	−3	−1	+2	+6	+6	−3	+1	+24	+27
170 E.	+1	−1	−5	−4	+3	+7	+5	−3	−2	−1	+2	+5	+5	−1	+3	+32	+28
180	−1	−3	−6	−3	+4	+8	+4	−3	−2	−1	+1	+3	+4	0	+5	+28	+28
170 W.	−1	−6	−7	−3	+3	+7	+4	−1	0	−2	+2	+2	+3	0	+7	+25	+25
160	0	−8	−8	−3	+2	+6	+3	0	0	−3	+2	+2	+2	+1	+6	+22	+23
150	+1	−8	−6	−2	0	+4	+3	+2	0	−2	+3	+2	+2	+2	+2	+17	+20
140	+3	−7	−5	0	0	+4	+2	+1	0	0	+1	+2	+2	+3	0	+13	+15
130	+6	−4	−3	+1	+1	+4	+2	+2	0	+1	+3	+2	+3	+3	−2	+6	+12
120	+7	−1	0	+3	+2	+3	+1	0	−1	+3	+3	+2	+4	+2	−2	−2	+13
110	+7	−1	+1	+4	+2	+2	+2	−1	−2	+2	+3	+3	+5	+2	−3	−5	+7
100	−1	−7	+1	+2	+3	+2	+3	−2	−3	0	+3	+4	+5	+3	−4	−7	+2
90	+1	−2	+2	0	+1	+2	+5	−2	−3	0	+4	+4	+6	+6	−2	−6	−2
80	−2	+1	+2	+1	+2	+2	+6	−2	−4	0	+2	+3	+4	+7	−1	−4	−6
70	−4	0	+2	+3	+2	+1	+4	+1	−3	−3	+2	+1	+3	+6	+2	−4	−9
60	−5	−5	0	+2	+1	0	+2	−1	−2	−2	0	+1	+2	+5	0	−5	−10
50	−5	−8	−2	−2	−1	+1	0	−2	−1	0	+4	+2	+1	+2	0	−5	−11
40	−3	−9	−2	−2	−1	+1	−1	−1	+1	0	+2	−1	+1	−5	−5	−7	−13
30	−6	−9	−1	−3	−1	+1	0	+1	−1	+1	+2	+1	+3	−8	−9	−9	−14
20	−7	−7	−1	−1	+1	+2	+3	+4	−4	−2	+1	+3	+4	−12	−13	−13	−15
10 W.	−8	−7	0	0	+1	+2	+5	+7	−5	−4	+1	+5	+4	−17	−19	−18	−16

TABLE XII

I, Inclination. Chart minus Computed. Unit 0°·1

	+80°	70°	60°	50°	40°	30°	20°	+10°	0°	−10°	20°	30°	40°	50°	60°	70°	−80°
0° E.	+6	+8	+3	+1	+4	−8	−4	−15	−14	0	+18	+27	+10	−20	−33	−28	−9
10	+5	+6	+2	−2	0	−7	−10	−15	−26	−3	+9	+11	+1	−27	−38	−31	−9
20	+5	+5	+1	−3	−1	−7	−8	−6	−10	−8	+5	+4	−12	−28	−38	−33	−7
30	+5	+6	+1	−5	−1	+1	−8	−8	−3	−11	0	−1	−10	−29	−37	−30	−6
40	+6	+5	+1	−5	−2	+1	−13	−10	−11	−12	−3	−5	−13	−27	−36	−26	−3
50	+5	+3	+1	−2	0	+1	−10	−5	−8	−7	−3	−3	−15	−27	−31	−23	+1
60	+3	+2	+1	−1	−6	+2	−2	−2	+9	+13	+4	+5	−11	−24	−29	−17	+2
70	+3	+1	−1	−2	−5	+1	−1	+2	+13	+26	+15	+15	0	−16	−22	−9	+5
80	+3	+2	−2	−4	−5	+1	+4	+5	+13	+27	+27	+27	+11	−8	−16	−8	+4
90	+3	+1	−3	−5	−9	+2	+5	+4	+13	+28	+32	+34	+20	−1	−13	−9	+2
100	+2	+1	−6	−2	−8	+1	+7	+8	+4	+20	+25	+33	+20	+4	−8	−11	+3
110	+1	+2	−6	+4	−2	−5	+3	+7	+14	+13	+19	+22	+21	+13	−7	−10	+5
120	−1	+2	−4	+8	+1	−2	−2	+4	+10	+12	+14	+15	+21	+17	−3	−8	+7
130	−3	+3	−1	+10	0	−1	−5	+7	+6	+11	+3	+10	+16	+15	+3	−1	+10
140	−4	+2	+7	+11	0	+4	−4	+4	+4	+7	+4	+6	+11	+11	+9	+5	+13
150	−4	+3	+13	+10	0	−4	−5	−1	+9	−1	+4	+4	+8	+6	+8	+14	+17
160	−3	+3	+16	+7	−1	−1	−7	−5	+10	−1	+4	+2	+7	+5	+3	+3	+19
170 E.	−2	+1	+14	+2	−4	−1	−6	−10	+8	−5	+5	0	+5	+3	+1	+17	+19
180	−1	+1	+11	+3	−4	−1	−3	−1	+12	−2	+3	−3	+2	+2	0	+17	+21
170 W.	−1	+5	+8	+2	−5	−4	−4	−3	+15	−1	+4	−1	0	+2	0	+17	+19
160	−1	+9	+6	+3	−5	−2	−2	0	+9	+2	+4	−1	+1	+5	+3	+17	+18
150	−1	+9	+7	+1	−1	−1	+1	+1	+5	+7	+2	0	+6	+12	+11	+16	+18
140	0	+7	+8	+1	+1	+1	+3	+4	+10	+3	+3	+4	+12	+14	+14	+20	+17
130	−1	+4	+4	0	+1	+2	+7	+5	+7	+9	+4	+7	+15	+14	+15	+20	+18
120	−1	+2	+3	+1	0	−1	+6	+7	+5	+8	+6	+9	+14	+10	+11	+14	+18
110	−2	0	−2	0	+3	−2	+4	+10	+2	+11	+9	+13	+13	+6	+8	+8	+17
100	+1	+6	−1	+1	+4	+3	+3	+8	+4	+15	+17	+17	+14	+6	+7	+7	+14
90	+1	+2	0	+1	+6	+5	+6	+2	0	+12	+22	+19	+14	+7	+1	+6	+10
80	+2	0	0	−1	+4	+3	0	+4	+3	+18	+20	+21	+17	+11	+4	+5	+7
70	+3	0	−3	−2	+2	+5	−1	+2	0	+27	+18	+22	+17	+13	+7	+6	+4
60	+4	+3	+1	+2	+3	+2	0	+6	+7	+27	+20	+15	+15	+12	+6	+6	+1
50	+4	+6	+4	+3	+4	+5	+9	+12	+22	+34	+18	+11	+12	+5	+3	+7	−2
40	+5	+7	+6	+5	+5	+7	+16	+22	+31	+28	+14	+16	+9	−2	−1	+3	−4
30	+5	+7	+7	+4	+8	+11	+19	+26	+30	+30	+30	+25	+15	−6	−10	−2	−6
20	+6	+7	+8	+3	+11	+10	+10	+19	+16	+21	+30	+35	+19	−6	−18	−10	−7
10 W.	+6	+7	+6	+2	+8	−2	+2	+6	+3	+14	+27	+35	+15	−14	−28	−20	−8

14. It seemed desirable to redetermine the coefficients, taking into account the spheroidal figure of the Earth. The formulae required for this analysis are given by J. C. Adams in Section VI, Part II of his *Collected Papers*, Vol. II. They can be summarized as follows:—

$$H'^m_n = \sin^m \theta' G'^m_n,$$

$$X_n^m = a^{n+2}[(n-m)\sin^{m+1}\theta' G'^{m+1}_n - m \sin^{m-1}\theta' \cos\theta' G'^m_n]/r^{n+2},$$

$$Y_n^m = a^{n+2} m \sin^{m-1}\theta' G'^m_n / r^{n+2},$$

$$Z_n^m = a^{n+2}(n+1)\sin^m \theta' G'^m_n / r^{n+2}.$$

The values of $\ln(a^n/r^n)$ for values of n from 1 to 12; of $\ln\cos\theta'$, of $\ln\sin^m\theta'$ for values of m from 1 to 10; and the values of G'^m_n are tabulated in Section II, Part II. r denotes the radius of the Earth and θ' the geocentric latitude.

From the values of X_n^m, Y_n^m, Z_n^m are computed the primed quantities defined by

$$X'^m_n = X_n^m \cos\psi + Z_n^m \sin\psi,$$

$$Y'^m_n = Y_n^m,$$

$$Z'^m_n = -X_n^m \sin\psi + Z_n^m \cos\psi,$$

ψ denotes the angle of the vertical; the values of $\ln\cos\psi$ and $\ln\sin\psi$ are tabulated by Adams. Unprimed quantities relate to axes along and at right angles to the radius vector; primed quantities to the normal and tangential components.

The equations

$$\Sigma X'^m_n g_n^m = x'_m,$$

$$\Sigma Y'^m_n g_n^m = y'_m,$$

$$\Sigma Z'^m_n g_n^m = z'_m,$$

with similar equations in h_n^m, are then used to determine the coefficients, the right-hand members being derived from the tabulated chart data for the north, east and vertical components, as in the solutions for the spherical Earth. The combining weights for the different latitude zones, given in Table IV, were again used, and a single solution combining the data from all three components of the field was made.

The values of the coefficients from the solution for the spheroidal Earth are given in the last column of Table V. The first six coefficients (g_n^0) are changed slightly, but the remaining coefficients are substantially unaltered. The solution does not add anything of special interest to the solution for the spherical Earth and it does not appear that the additional labour involved in the solution for the spheroidal Earth is justified in future harmonic analyses.

15. Comparison between the principal coefficients and those determined from other spherical harmonic analyses, including two determinations of slightly later epoch, are given in Table XIII. The coefficients g_1^0, g_2^0, g_2^2 and h_2^1 appear to change linearly with the time, h_2^1 having the most rapid rate of change. The change in g_1^1 appears not to be linear and is slower now than a century ago. The values of g_2^1 and h_1^1 show a considerable scatter but no appreciable change with time is apparent.

The values of the other coefficients have been compared with their values as found by Adams for the epochs 1845 and 1880, by Dyson and Furner for 1922, and by Vestine and others for 1945. The coefficients of a number of the higher harmonic terms show appreciable changes in the course of a century; g_5^0, g_3^1, g_4^1, h_5^1, g_4^2, g_5^2, g_6^2, h_6^2 are those that show the largest and most systematic changes. The fact that many of the coefficients change so rapidly is an indication of the complicated structure of the secular change field.

TABLE XIII

Values of principal coefficients

		g_1^0	g_2^0	g_1^1	g_2^1	g_2^2	h_1^1	h_2^1	h_2^2
Erman-Petersen	1829	+3 201	+ 12	+284	−445	+ 12	−601	+ 7	−127
Gauss	1835	3 235	− 76	311	505	+ 2	625	− 21	136
Adams	1845	3 219	− 13	278	491	− 4	578	+ 18	116
Adams	1880	3 168	+ 73	243	514	53	603	129	129
Schmidt	1885	3 168	75	222	481	56	595	123	129
Fritsche	1885	3 164	53	241	495	59	591	130	123
Dyson & Furner	1922	3 095	133	226	518	125	592	215	73
Jones & Melotte	1942	3 039	176	218	509	135	555	260	44
Afanasieva	1945	3 032	187	229	498	130	590	253	42
Vestine & Lange	1945	+3 057	+190	+210	−512	−141	−581	+287	− 46

16. The first-order harmonic, which corresponds to the dipole field, is given by

$$g_1^0 \sin\lambda + (g_1^1 \cos\phi + h_1^1 \sin\phi)\cos\lambda.$$

Writing $\quad g_1^0 = H_0 \cos\theta_0;\ g_1^1 = H_0 \sin\theta_0 \cos\phi_0;\ h_1^1 = H_0 \sin\theta_0 \sin\phi_0,$

H_0 gives the intensity of the dipole field; θ_0, ϕ_0 are the colatitude and the longitude of the northern pole of the dipole field, or the northern geomagnetic pole. The southern geomagnetic pole is the antipodal point to the northern: these poles are to be distinguished from the magnetic dip poles.

The values of H_0, θ_0 and ϕ_0 are given in Table XIV. The positions of the two poles from the present analysis are 78°·9 N., 68°·5 W., and 78°·9 S., 111°·5 E.

TABLE XIV

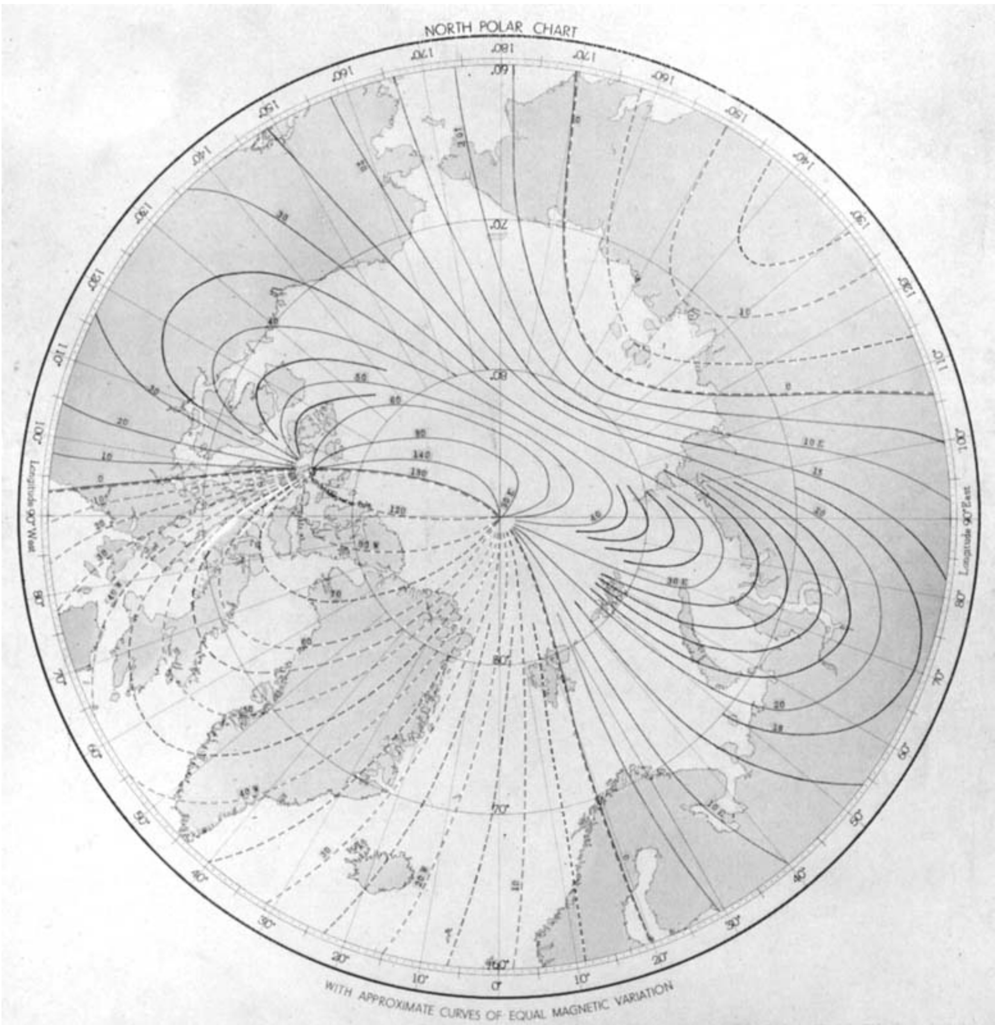
The Dipole Field

	Epoch	H_0	θ_0	ϕ_0
			°	°
Erman-Petersen	1829	·326 9	11·7	−64·7
Gauss	1835	·330 9	12·1	63·5
Adams	1845	·328 2	11·2	64·3
Adams	1880	·323 4	11·6	68·0
Schmidt	1885	·323 1	11·3	69·5
Fritsche	1885	·322 8	11·4	67·8
Dyson & Furner	1922	·315 9	11·6	69·1
Jones & Melotte	1942	·309 7	11·1	68·5
Afanasieva	1945	·309 7	11·1	68·8
Vestine & Lange	1945	·311 9	11·4	70·0

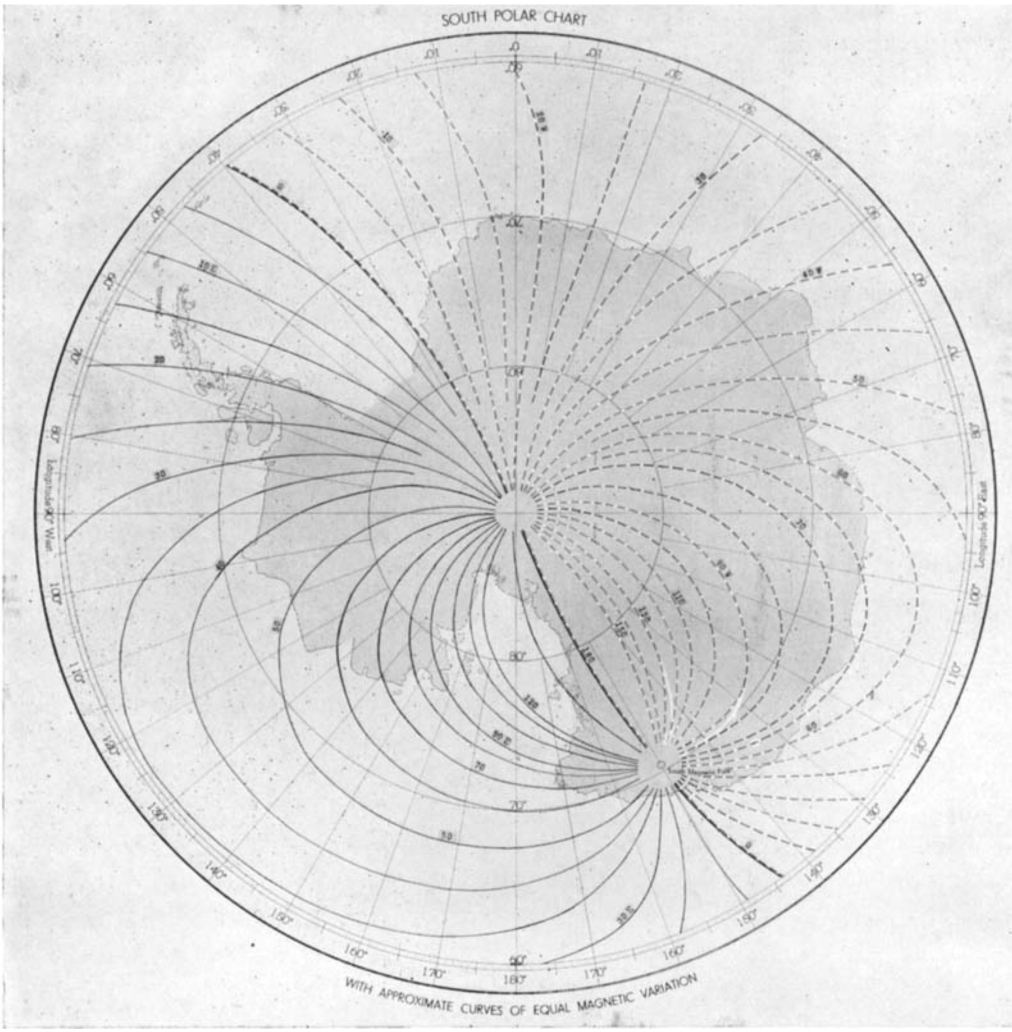
The values of H_0, θ_0, ϕ_0 can be represented by the formulae:

$$H_0 = 0{\cdot}3187 - 0{\cdot}0170\ (T-1900),$$
$$\theta_0 = 11^\circ{\cdot}4 - 0^\circ{\cdot}4\ (T-1900),$$
$$-\phi_0 = 67^\circ{\cdot}8 + 4^\circ{\cdot}5\ (T-1900),$$

where T is measured in centuries.



Computed isogonals for north polar regions



Computed isogonals for south polar region

Sir Harold Spencer Jones and P. J. Melotte, The harmonic analysis of the Earth's magnetic field, for epoch 1942

It will be seen that during the period covered by these analyses the intensity of the dipole field has progressively decreased, the rate being about 5 per cent per century. The latitudes of the geomagnetic poles have not changed appreciably, but the northern pole shows a progressive movement westwards, and the southern pole a corresponding movement, of 4°·5 of longitude per century.

17. The geomagnetic poles, on the dipole axis, are not the same as the magnetic dip poles, the two points on the Earth's surface at which the horizontal intensity vanishes and the inclination or dip has the value of 90°. In the construction of the charts, the positions adopted in the 1942 charts for the north and south magnetic poles were 71° N., 96° W.; 71½° S., 151° E. respectively. The position of the north magnetic pole is practically the position determined by Amundsen from observations in the Boothia Peninsula between 1903 November and 1905 May and is within one degree of the position assigned by Ross in 1831, the agreement suggesting that the movement of the pole is slight.

In the course of the preparation of the *Admiralty Magnetic Charts for 1922*, a complete revision of the north polar area was made by one of us, by constructing first the projected lines of magnetic force.* The following comment was made:

> "It will be seen that the revision has considerably reduced the residuals which, however, are still systematic in their nature. It does not seem possible to reduce them further whilst adhering to the position which has been adopted for the magnetic pole. A better fit could have been obtained if a position about 2° further north had been adopted."

Nevertheless, as there had been no observations in the vicinity of the north magnetic pole, the Amundsen position continued to be used, not only in the British charts but in those of other countries.

The south magnetic pole has never been reached and there has been no detailed survey round it. The position assigned for the 1922 and 1942 charts was based on the observations in the vicinity of the magnetic pole obtained by Shackleton's expedition of 1907–09, and Mawson's expedition of 1911–14.

18. Charts for the polar areas of the different elements were constructed by computing the values of the elements at intervals of 2½° of latitude and of 10° of longitude, from latitude 67½° to the pole; from these charts the positions of the magnetic poles were derived both for a spherical and for a spheroidal Earth. Positions were assigned by Dyson and Furner from their analysis of the 1922 charts, but no information is given as to how they were derived.

The comparison between the assumed and the computed positions is as follows:—

	N. Mag. Pole	S. Mag. Pole
Assumed (observations *c.* 1910)	71° N., 96° W.	71½° S., 151° E.
Dyson and Furner (1922)	75° N., 100° W.	71° S., 151° E.
Jones and Melotte (1942)		
Spherical Earth	77° N., 103½° W.	71° S., 150½° E.
Spheroidal Earth	76° N., 102° W.	70° S., 150° E.

The differences between the assumed and computed position of the north magnetic pole seemed to be appreciably larger than would be expected from the general uncertainty of the chart data, while the comparison with the determination by Dyson and Furner suggested that this pole had moved in a direction

* H. Spencer Jones, *Geographical Journal*, **62**, 419, 1923.

slightly to the west of north since its position was determined by Amundsen. For the south magnetic pole the assumed and computed positions were in as good agreement as could be expected from the uncertainties attaching to both of them.

19. When plans for the north polar flights of the Lancastrian aircraft *Aries* from the Empire Air Navigation School, Shawbury, were under consideration in 1945, it was therefore suggested that a flight should be made over both the Amundsen position of the north magnetic pole and the computed position. The data provided by two flights in May 1945 indicated that the true position of the pole was probably between these two positions but nearer to the computed position; the inference was that the position of the north magnetic pole in 1945 was approximately 74° N., 100° W. The flights provided the first definite information that the pole had moved considerably northwards since Amundsen's observations in 1904. Since that date, independent confirmatory evidence has been provided by ground observations in the Canadian Eastern Arctic, arranged by the Dominion Observatory, Ottawa. In a report made in 1947 May to the Royal Astronomical Society of Canada, R. Glen Madill stated that the Dominion Observatory had for many years been fully aware that the magnetic pole was travelling in a northerly direction, this conclusion being based on the convergence of the isogonals based on observations made periodically at a number of repeat stations extending from Newfoundland to Alaska. During recent years the network of stations occupied by observers from the Dominion Observatory has been extended considerably northwards. In 1946 observations were made in Denmark Bay, Victoria Island, and Fort Ross, Somerset Island; during the summer of 1947 a full-scale airborne expedition made extensive observations at ten stations throughout the Northwest Territories, six of which were on the islands in the vicinity of the north magnetic pole. The mean position of the magnetic pole derived from these observations is $73\frac{1}{2}$° N., 100° W. This position will be adopted in the Admiralty magnetic charts for 1955 which are now being prepared at the Royal Greenwich Observatory.

20. In each of the polar regions the system of isogonals possesses two singular points, the geographical and magnetic dip poles. The structure of the isogonals is of some complexity and very few observational data have been obtained in recent years. The isogonals can best be constructed by computation from the harmonic analysis. In the two plates the isogonals derived in this way are depicted for the north and south polar areas. The positions adopted for the magnetic dip poles are the computed positions; though for the north polar area the computed position and the observed position are somewhat discordant, the charts can serve as a guide in future magnetic chart construction, being adjusted where necessary as observational data become available.

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1952 *April* 29.